

**Grade Four Mathematics Content Standards**

By the end of grade four, students understand large numbers and addition, subtraction, multiplication, and division of whole numbers. They describe and compare simple fractions and decimals. They understand the properties of, and the relationships between, plane geometric figures. They collect, represent, and analyze data to answer questions.

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**Number Sense**

**1.0 Students understand the place value of whole numbers and decimals to two decimal places and how whole numbers and decimals relate to simple fractions. Students use the concepts of negative numbers:**

**1.1** Read and write whole numbers in the millions.

Which of these is the number 5,005,014?

- A five million, five hundred, fourteen
- B five million, five thousand, fourteen
- C five thousand, five hundred, fourteen
- D five billion, five million, fourteen (CST released test question, 2004)

**1.2** Order and compare whole numbers and decimals to two decimal places.

Which is bigger: 3.1 or 3.09?

**1.3** Round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand, or hundred thousand.

1548 Two hundred twenty-four students attend Green Street School. Round  
1549 this number to the nearest hundred.

1550 Lunch was served to 3,778 students. Round this number to the  
1551 nearest thousand.

1552 Each year it is estimated that 42,225 Canadian geese migrate south to  
1553 warmer climates. Round this number to the nearest ten thousand.

1554 **1.4** Decide when a rounded solution is called for and explain why such a  
1555 solution may be appropriate.

1556 Norberto has ten dollars and he wants to buy some ballpoints, which  
1557 cost \$2.35; some notebooks, which cost \$4.40; and a fancy eraser,  
1558 which costs \$1.45. He wants to make sure he has enough money to  
1559 pay for all of them, so he rounds the cost of each item to the nearest  
1560 dollar and adds them up:  $\$2 + \$4 + \$1 = \$7$ . He concludes that his ten  
1561 dollars would be sufficient to buy all the items. Is he correct and, if so,  
1562 why? If the estimate that he makes turns out to be \$8 instead of \$7,  
1563 should he be concerned?

1564 1.5 Explain different interpretations of fractions, for example, parts of a  
1565 whole, parts of a set, and division of whole numbers by whole  
1566 numbers; explain equivalence of fractions (see Standard 4.0).

1567 1.6 Write tenths and hundredths in decimal and fraction notations and  
1568 know the fraction and decimal equivalents for halves and fourths (e.g.,  
1569  $\frac{1}{2} = 0.5$  or  $0.50$ ;  $\frac{7}{4} = 1\frac{3}{4} = 1.75$ ).

1570 Which fraction means the same as 0.17? (CST released test question,  
1571 2004)

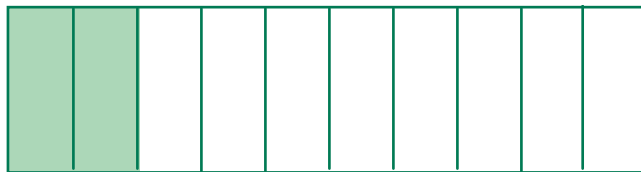
1572                       $\frac{17}{10}$                        $\frac{17}{100}$                        $\frac{17}{1000}$                        $\frac{17}{1}$

1573                      1.7    Write the fraction represented by a drawing of parts of a figure;  
 1574    represent a given fraction by using drawings; and relate a fraction to a  
 1575    simple decimal on a number line.

1576

1577

1578                      Which number represents the shaded part of the figure? (Adapted



1579                      from TIMSS gr. 4, M-5)

1580                      1. 2.8    2. 0.5

1581                      3. 0.2    4. 0.02

1582                      **1.8**    Use concepts of negative numbers (e.g., on a number line, in counting,  
 1583    in temperature, in “owing”).

1584                      Yesterday’s temperature was 5 degrees Celsius, but the temperature  
 1585    dropped 9 degrees Celsius overnight. What is today’s temperature?

1586                      Determine if the following number sentences are true or false by  
 1587    identifying the relative positions of each number on a number line:

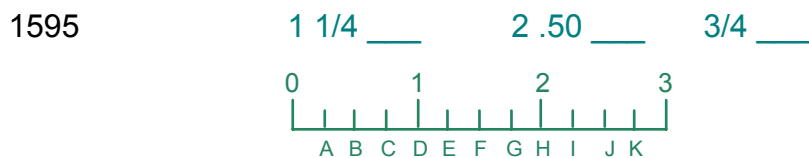
1588     $-9 > -10$      $-31 < -29$

1589                      **1.9**    Identify on a number line the relative position of positive fractions,  
 1590    positive mixed numbers, and positive decimals to two decimal places.

1591 Write a positive number for each letter on the number line shown  
 1592 below. Which letter represents the number closest to 2.5?



1593 Write the letter that represents where each number would go on the  
 1594 number line shown below:



1596 Determine if the following number sentences are true or false by  
 1597 identifying the relative positions of each number on a number line:

1598 1.  $\frac{1}{4} > 2.54$

1599 2.  $\frac{5}{2} < 2.6$

1600 3.  $\frac{12}{18} = \frac{2}{3}$  (Note the equivalence of fractions.)

1601 4.  $\frac{4}{5} < \frac{13}{15}$

1602 **2.0 Students extend their use and understanding of whole numbers to the**  
 1603 **addition and subtraction of simple decimals:**

1604 2.1 Estimate and compute the sum or difference of whole numbers and  
 1605 positive decimals to two places.

1606 Solve  $55.73 - 48.25 = ?$

1607 2.2 Round two-place decimals to one decimal or the nearest whole  
 1608 number and judge the reasonableness of the rounded answer.

In her science class, Li Ping weighs two samples of quartz and determines that the first has a weight of 3.44 grams and the second has a weight of 2.39 grams. Her teacher wants Li Ping to report the combined weight of the two samples to the nearest tenth of a gram, and the nearest gram, however the scale cannot measure weights over 5 grams. Li Ping decides to round the numbers first, then add them.

Is  $3.4 + 2.4$  a reasonable estimate of the combined weights, to the nearest tenth of a gram?

Is  $3 + 2$  a reasonable estimate of the combined weights, to the nearest gram?

**3.0 Students solve problems involving addition, subtraction, multiplication, and division of whole numbers and understand the relationships among the operations:**

**3.1** Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multidigit numbers.

*Solve these problems using the standard algorithms:*

$$619,581 - 23,183 = ?$$

$$6,747 + 321,105 = ?$$

**3.2** Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multidigit number by a two-digit number and for dividing a multidigit number by a one-digit number; use relationships between them to simplify computations and to check results.

Singh and Sepideh work independently to solve the problem  $783 \times 23 = ?$ . They apply slightly different approaches, as shown below. Explain why both approaches are valid and give the same answer.

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783	783	2,349
<u>   x3   </u>	<u>   x20   </u>	<u>+15,660</u>
2,349	15,660	18,009
Singh		

783
<u>   x23   </u>
2,349
<u>+15,660</u>
18,009
Sepideh

1645

1646

1647

**3.3**

Solve problems involving multiplication of multidigit numbers by two-digit numbers.

1648

1649

**3.4**

Solve problems involving division of multidigit numbers by one-digit numbers.

1650

1651

Solve each of the following problems and observe the different roles played by the number 37 in each situation:

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1. Four children shared 37 dollars equally. How much did each get?

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2. Four children shared 37 pennies as equally as possible. How many pennies did each get?

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3. Cars need to be rented for 37 children going on a field trip. Each car can take 12 children in addition to the driver. How many cars must be rented?

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There are 9 rows of seats in a theater. Each row has the same number of seats. If there is a total of 162 seats, how many seats are in each row? (CST released test question, 2004)

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1661

1662

#### **4.0 Students know how to factor small whole numbers:**

1663

4.1 Understand that many whole numbers break down in different ways (e.g.,  $12 = 4 \times 3 = 2 \times 6 = 2 \times 2 \times 3$ ).

1664

1665 In how many distinct ways can you write 60 as a product of two  
 1666 numbers?

1667 4.2 Know that numbers such as 2, 3, 5, 7, and 11 do not have any factors  
 1668 except 1 and themselves and that such numbers are called prime  
 1669 numbers.

1670  
 1671 Circle all of the prime numbers in these different representations of 24:

1672  $2 \times 12$   $3 \times 8$   $4 \times 6$   $2 \times 2 \times 6$   $2 \times 3 \times 4$   $2 \times 2 \times 2 \times 3$   $1 \times 24$

1673

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## 1674 Algebra and Functions

1675 **1.0 Students use and interpret variables, mathematical symbols, and**  
 1676 **properties to write and simplify expressions and sentences:**

1677 1.1 Use letters, boxes, or other symbols to stand for any number in simple  
 1678 expressions or equations (e.g., demonstrate an understanding and the  
 1679 use of the concept of a variable).

1680 Tanya has read the first 78 pages of a 130-page book. Give the  
 1681 number sentence that can be used to find the number of pages Tanya  
 1682 must read to finish the book. (Adapted from TIMSS gr. 4, I-7)

1683 1.  $130 + 78 = \underline{\hspace{2cm}}$

1684 2.  $\underline{\hspace{2cm}} - 78 \square = 130$

1685 3.  $130 - 78 = \underline{\hspace{2cm}}$

1686 4.  $130 - \underline{\hspace{2cm}} = 178$

1687 **1.2** Interpret and evaluate mathematical expressions that now use  
 1688 parentheses.

1689 Evaluate the two expressions:

1690  $(28 - 10) - 8 = \underline{\hspace{2cm}}$  and  $28 - (10 - 8) = \underline{\hspace{2cm}}$ . (CST released test

- 1691 question, 2004)
- 1692 What is the value of the expression below?
- 1693  $(13 + 4) - (7 \times 2)$
- 1694 **1.3** Use parentheses to indicate which operation to perform first when
- 1695 writing expressions containing more than two terms and different
- 1696 operations.
- 1697
- 1698 Solve  $5 \times (8-2) = ?$  (CST released test question, 2004)
- 1699 1.4 Use and interpret formulas (e.g., area = length  $\times$  width or  $A = lw$ ) to
- 1700 answer questions about quantities and their relationships.
- 1701 Vik has a car that has a 16 gallon gas tank, and when it is filled he can
- 1702 drive 320 miles before running out of gas. How can Vik calculate his
- 1703 car's mileage, in miles/gallon?
- 1704 **1.5** Understand that an equation such as  $y = 3x + 5$  is a prescription for
- 1705 determining a second number when a first number is given.
- 1706 **2.0** Students know how to manipulate equations:
- 1707 **2.1** Know and understand that equals added to equals are equal.
- 1708 I.
- 1709 The letters S and T stand for numbers. If  $S - 100 = T - 100$ , which statement is true?
- $S = T$        $S > T$        $S = T + 100$        $S > T + 100$
- 1710 (CST released test question, 2004)
- 1711 **2.2** Know and understand that equals multiplied by equals are equal.
- 1712 What number goes into the box to make this number sentence true?
- 1713  $(7-3) \times 5 = 4 \times [\text{box}]$  (CST released test question, 2004)
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1715 **Measurement and Geometry**

1716 **1.0 Students understand perimeter and area:**

1717 1.1 Measure the area of rectangular shapes by using appropriate units,  
1718 such as square centimeter ( $\text{cm}^2$ ), square meter ( $\text{m}^2$ ), square kilometer  
1719 ( $\text{km}^2$ ), square inch ( $\text{in}^2$ ), square yard ( $\text{yd}^2$ ), or square mile ( $\text{mi}^2$ ).

1720 1.2 Recognize that rectangles that have the same area can have different  
1721 perimeters.

1722 Draw a rectangle whose area is 120 and whose perimeter exceeds 50.  
1723 Draw another rectangle with the same area whose perimeter exceeds  
1724 240.

1725 1.3 Understand that rectangles that have the same perimeter can have  
1726 different areas.

1727 Is the area of a  $45 \times 55$  rectangle (in  $\text{cm}^2$ ) smaller or bigger than that  
1728 of a square with the same perimeter?

1729 Draw a rectangle whose perimeter is 40 and whose area is less than  
1730 20.

1731 1.4 Understand and use formulas to solve problems involving perimeters  
1732 and areas of rectangles and squares. Use those formulas to find the  
1733 areas of more complex figures by dividing the figures into basic  
1734 shapes.

1735 The length of a rectangle is 6 cm, and its perimeter is 16 cm. What is  
1736 the area of the rectangle in square centimeters? (Adapted from TIMSS  
1737 gr. 8, K–5)

1738 **2.0 Students use two-dimensional coordinate grids to represent points**  
1739 **and graph lines and simple figures:**

1740 **2.1** Draw the points corresponding to linear relationships on graph paper  
1741 (e.g., draw 10 points on the graph of the equation  $y = 3x$  and connect  
1742 them by using a straight line).

1743 Draw ten points on the graph of the equation  $x = 4$ .

1744 Draw ten points on the graph of the equation  $y = 71$ .

1745 Draw ten points on the graph of the equation  $y = 2x + 4$ .

1746 **2.2** Understand that the length of a horizontal line segment equals the  
1747 difference of the  $x$ -coordinates.

1748 What is the length of the line segment joining the points  $(6, -4)$  and  
1749  $(21, -4)$ ?

1750 **2.3** Understand that the length of a vertical line segment equals the  
1751 difference of the  $y$ -coordinates.

1752 What is the length of the line segment joining the points  $(121, 3)$  to  
1753  $(121, 17)$ ?

1754 **3.0 Students demonstrate an understanding of plane and solid geometric**  
1755 **objects and use this knowledge to show relationships and solve**  
1756 **problems:**

1757 3.1 Identify lines that are parallel and perpendicular.

1758 (Teachers are advised to introduce the terms “intersecting lines” and  
1759 “non-intersecting lines” when dealing with this standard.)

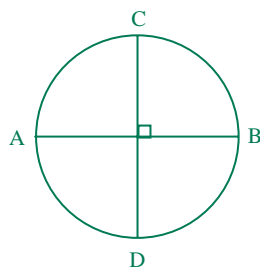
1760 3.2 Identify the radius and diameter of a circle.

1761 3.3 Identify congruent figures.

1762 3.4 Identify figures that have bilateral and rotational symmetry.

1763 Craig folded a piece of paper in half and cut out a shape along the  
1764 folded edge. Draw a picture to show what the cutout shape will look  
1765 like when it is opened up and flattened out (Adapted from TIMSS gr. 4,  
1766 T-5).

1767 Let  $AB$ ,  $CD$  be perpendicular diameters of a circle, as shown. If we  
1768 reflect across the line segment  $CD$ , what happens to  $A$  and what  
1769 happens to  $B$  under this reflection?



1770 3.5 Know the definitions of a right angle, an acute angle, and an obtuse  
1771 angle. Understand that  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ , and  $360^\circ$  are associated,  
1772 respectively, with  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$ , and full turns.

1773 3.6 Visualize, describe, and make models of geometric solids (e.g.,  
1774 prisms, pyramids) in terms of the number and shape of faces, edges,  
1775 and vertices; interpret two-dimensional representations of three-  
1776 dimensional objects; and draw patterns (of faces) for a solid that, when  
1777 cut and folded, will make a model of the solid.

1778 3.7 Know the definitions of different triangles (e.g., equilateral, isosceles,  
1779 scalene) and identify their attributes.

1780 Name each of the following triangles:

1781 1. No equal sides

1782 2. 2 equal sides

1783 3. 3 equal sides

1784 3.8 Know the definition of different quadrilaterals (e.g., rhombus, square,  
1785 rectangle, parallelogram, trapezoid).

1786 Explain which of the following statements are true and why.

1787 1. All squares are rectangles.

1788 2. All rectangles are squares.

1789 3. All parallelograms are rectangles.

1790 4. All rhombi are parallelograms.

1791 5. Some parallelograms are squares.

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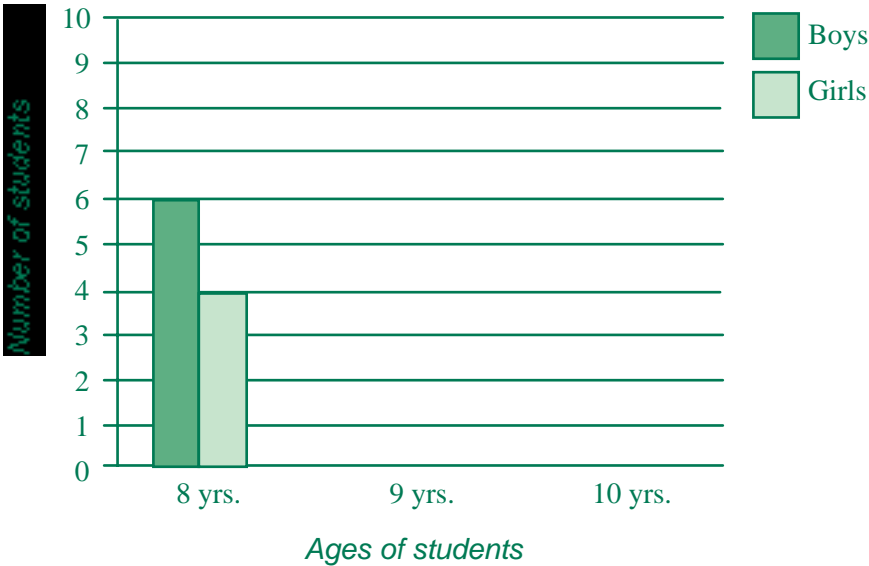
## 1792 **Statistics, Data Analysis, and Probability**

1793 **1.0 Students organize, represent, and interpret numerical and categorical**  
1794 **data and clearly communicate their findings:**

1795 The following table shows the ages of the girls and boys in a club. Use  
1796 the information in the table to complete the graph for ages 9 and 10.

1797 (Adapted from TIMSS gr. 4, S-1)

Ages	Number of Girls	Number of Boys
8	4	6
9	8	4
10	6	10



1798 1.1 Formulate survey questions; systematically collect and represent data  
1799 on a number line; and coordinate graphs, tables, and charts.

1800 1.2 Identify the mode(s) for sets of categorical data and the mode(s),  
1801 median, and any apparent outliers for numerical data sets.

1802 1.3 Interpret one- and two-variable data graphs to answer questions about  
1803 a situation.

1804 **2.0 Students make predictions for simple probability situations:**

1805 Nine identical chips numbered 1 through 9 are put in a jar. When a  
1806 chip is drawn from the jar, what is the probability that it has an even  
1807 number? (Adapted from TIMSS gr. 8, N-18)

- 1808           2.1   Represent all possible outcomes for a simple probability situation in an  
1809                   organized way (e.g., tables, grids, tree diagrams).
- 1810           2.2   Express outcomes of experimental probability situations verbally and  
1811                   numerically (e.g., 3 out of 4;  $\frac{3}{4}$ ).
- 1812                   Royce has a bag with 8 red marbles, 4 blue marbles, 5 green marbles,  
1813                   and 9 yellow marbles all the same size. If he pulls out 1 marble without  
1814                   looking, which color is he most likely to choose? (CST released test  
1815                   question, 2004)
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1816   **Mathematical Reasoning**

1817   **1.0   Students make decisions about how to approach problems:**

- 1818           1.1   Analyze problems by identifying relationships, distinguishing relevant  
1819                   from irrelevant information, sequencing and prioritizing information,  
1820                   and observing patterns.
- 1821           1.2   Determine when and how to break a problem into simpler parts.

1822   **2.0   Students use strategies, skills, and concepts in finding solutions:**

- 1823           2.1   Use estimation to verify the reasonableness of calculated results.
- 1824           2.2   Apply strategies and results from simpler problems to more complex  
1825                   problems.
- 1826           2.3   Use a variety of methods, such as words, numbers, symbols, charts,  
1827                   graphs, tables, diagrams, and models, to explain mathematical  
1828                   reasoning.

- 1829           2.4   Express the solution clearly and logically by using the appropriate  
1830                   mathematical notation and terms and clear language; support  
1831                   solutions with evidence in both verbal and symbolic work.
- 1832           2.5   Indicate the relative advantages of exact and approximate solutions to  
1833                   problems and give answers to a specified degree of accuracy.
- 1834           2.6   Make precise calculations and check the validity of the results from the  
1835                   context of the problem.
- 1836   **3.0   Students move beyond a particular problem by generalizing to other**  
1837   **situations:**
- 1838   3.1   Evaluate the reasonableness of the solution in the context of the original  
1839           situation.
- 1840   3.2   Note the method of deriving the solution and demonstrate a conceptual  
1841           understanding of the derivation by solving similar problems.
- 1842   3.3   Develop generalizations of the results obtained and apply them in other  
1843           circumstances.

1844 **Grade Five Mathematics Content Standards**

1845 By the end of grade five, students increase their facility with the four basic  
1846 arithmetic operations applied to fractions and decimals and learn to add and  
1847 subtract positive and negative numbers. They know and use common measuring  
1848 units to determine length and area and know and use formulas to determine the  
1849 volume of simple geometric figures. Students know the concept of angle  
1850 measurement and use a protractor and compass to solve problems. They use  
1851 grids, tables, graphs, and charts to record and analyze data.

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1852 **Number Sense**

1853 **1.0 Students compute with very large and very small numbers, positive**  
1854 **integers, decimals, and fractions and understand the relationship**  
1855 **between decimals, fractions, and percents. They understand the**  
1856 **relative magnitudes of numbers:**

1857 1.1 Estimate, round, and manipulate very large (e.g., millions) and very  
1858 small (e.g., thousandths) numbers.

1859 **1.2** Interpret percents as a part of a hundred; find decimal and percent  
1860 equivalents for common fractions and explain why they represent the  
1861 same value; compute a given percent of a whole number.

1862 What is 40% of 250? (CST released test question, 2004)

1863 A test had 48 problems. Joe got 42 correct.

1864 1. What percent were correct?

1865 2. What percent were wrong?

1866 3. If Moe got 93.75% correct, how many problems did he get correct?



1867 1.3 Understand and compute positive integer powers of nonnegative  
 1868 integers; compute examples as repeated multiplication.

1869 Which is bigger:  $3^5$  or  $5^3$ ?

1870 **1.4** Determine the prime factors of all numbers through 50 and write the  
 1871 numbers as the product of their prime factors by using exponents to  
 1872 show multiples of a factor (e.g.,  $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$ ).

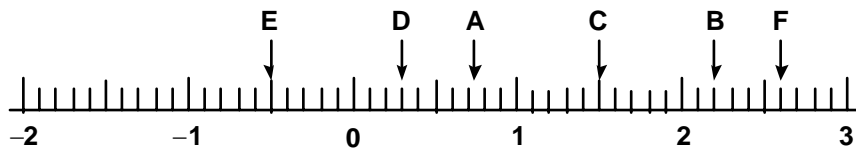
1873 Find the prime factorization of 48 and use exponents where  
 1874 appropriate.

1875 **1.5** Identify and represent on a number line decimals, fractions, mixed  
 1876 numbers, and positive and negative integers.

1877 Next to each number, write the letter that represents the quantity on  
 1878 the number line.

1879 2.2 \_\_\_\_\_ 0.3 \_\_\_\_\_ -0.5 \_\_\_\_\_

1880  $2\frac{6}{10}$  \_\_\_\_\_  $\frac{75}{100}$  \_\_\_\_\_ 1.5 \_\_\_\_\_



1881 Place the following numbers, in approximate positions, on the number  
 1882 line:  $1\frac{3}{7}$ , 1.43,  $\frac{23}{14}$ .

1883 **2.0** Students perform calculations and solve problems involving addition,  
 1884 subtraction, and simple multiplication and division of fractions and  
 1885 decimals:

**2.1**

Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.

Determine the following numbers:

1.  $11 + (-23)$

2.  $(-15) - 128$

3.  $51 - 24.7$

4.  $8.2 \times 24.7$

5.  $68.13 \div 3$

**2.2**

Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.

Find the quotient: 6 divided by 0.025

$15.12 \div 2.4 = ?$  (CST released test question, 2004)

**2.3**

Solve simple problems, including ones arising in concrete situations, involving the addition and subtraction of fractions and mixed numbers (like and unlike denominators of 20 or less), and express answers in the simplest form.

Suppose a galleon is a type of money worth 17 sickles. If Ludo borrows  $2 \frac{3}{17}$  galleons from Harry, then gives him back 12 sickles, how many galleons and sickles does he still owe?

Sally is training to walk in a marathon. In her second week of training, she walked  $5 \frac{3}{4}$  miles on Tuesday,  $5 \frac{1}{6}$  miles on Thursday, and  $16 \frac{3}{8}$  miles on Sunday. How many miles altogether did Sally walk on those three days?

1910 Jerry and Larry both ordered personal-sized pizzas for lunch. Jerry ate  
 1911  $\frac{3}{4}$  of his pizza, and Larry ate  $\frac{2}{3}$  of his pizza. Who ate more pizza and  
 1912 how much more did he eat?

1913 Given the following three pairs of fractions ( $\frac{3}{8}$  and  $\frac{1}{6}$ ,  $5\frac{3}{4}$  and  $2\frac{1}{3}$ ,  
 1914  $16$  and  $12\frac{7}{8}$ ), find for each pair its:

1915 1. Sum

1916 2. Difference

1917 2.4 Understand the concept of multiplication and division of fractions.

1918  $\frac{3}{4} \div \frac{3}{5} = \frac{?}{-}$  (CST released test question, 2004)

1919 2.5 Compute and perform simple multiplication and division of fractions  
 1920 and apply these procedures to solving problems.

1921 Given the following three pairs of fractions ( $\frac{3}{8}$  and  $\frac{1}{6}$ ,  $5\frac{3}{4}$  and  $2\frac{1}{3}$ ,  
 1922  $16$  and  $12\frac{7}{8}$ ), find for each pair its:

1923 1. Product

1924 2. Quotient in simplest terms

1925 Ericka has  $3\frac{1}{2}$  yards of cloth to make shirts. Each shirt requires  $\frac{7}{8}$   
 1926 yard. How many shirts can she make? How much cloth will she have  
 1927 left over?

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1928 **Algebra and Functions**

1929 **1.0 Students use variables in simple expressions, compute the value of**  
 1930 **the expression for specific values of the variable, and plot and**  
 1931 **interpret the results:**

1932 1.1 Use information taken from a graph or equation to answer questions  
 1933 about a problem situation.

1934 **1.2** Use a letter to represent an unknown number; write and evaluate  
 1935 simple algebraic expressions in one variable by substitution.

1936 If  $x$  is a number that satisfies  $3x + 2 = 14$ , can  $x$  be equal to 3?

1937 If  $N=4$ , what is the value of  $6 \times N-3$ ? (CST released test question,  
 1938 2004)

1939 1.3 Know and use the distributive property in equations and expressions  
 1940 with variables.

1941 What value for  $z$  makes this equation true?

1942  $8 \times 37 = (8 \times 30) + (8 \times z)$  (CST released test question, 2004)

1943 **1.4** Identify and graph ordered pairs in the four quadrants of the coordinate  
 1944 plane.

1945 Plot these points on a coordinate plane:  $(1, 2)$ ,  $(-4, -3)$ ,  $(12, -1)$ ,  $(0, 4)$ ,  
 1946  $(-4, 0)$ .

1947 **1.5** Solve problems involving linear functions with integer values; write the  
 1948 equation; and graph the resulting ordered pairs of integers on a grid.

1949 Which equation could have been used to create this function table?

1950 (CST released test question, 2004)

x	y
-9	-5
-2	2
4	8
11	15

1951

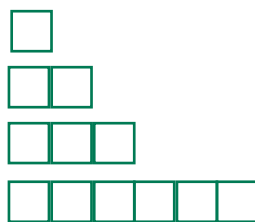
$$y = x/2 \quad y = 2x \quad y = x - 4 \quad y = x + 4$$

1952

One can build rows of squares with toothpicks, as shown below for the

1953

case of 1, 2, 3, and 6 squares, respectively:



1954

Explain why the following formula

1955

$$y = 3n + 1$$

1956

for the number of toothpicks  $y$  needed to form a row of  $n$  squares is

1957

correct. Graph this equation on a grid, and remember that  $n$  takes on

1958

only whole number values 1, 2, 3, 4, . . .

1959

1960

## Measurement and Geometry

1961

### 1.0 Students understand and compute the volumes and areas of simple

1962

objects:

1963

**1.1**

Derive and use the formula for the area of a triangle and of a

1964

parallelogram by comparing it with the formula for the area of a

1965

rectangle (i.e., two of the same triangles make a parallelogram with

1966

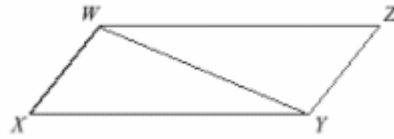
twice the area; a parallelogram is compared with a rectangle of the

1967

same area by pasting and cutting a right triangle on the parallelogram).

1968

In the figure below, WXYZ is a parallelogram.



1969

1970

If the area of triangle WXY is 22 square inches, what is the area of WXYZ? (CST released test question, 2004)

1971

1972

**1.2**

Construct a cube and rectangular box from two-dimensional patterns and use these patterns to compute the surface area for these objects.

1973

1974

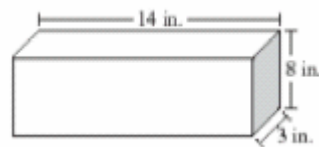
**1.3**

Understand the concept of volume and use the appropriate units in common measuring systems (i.e., cubic centimeter [ $\text{cm}^3$ ], cubic meter [ $\text{m}^3$ ], cubic inch [ $\text{in.}^3$ ], cubic yard [ $\text{yd.}^3$ ]) to compute the volume of rectangular solids.

1975

1976

1977



1978

1979

This rectangular prism has a length of 14 inches, a height of 8 inches, and a width of 3 inches. What is the volume? (CST released test question, 2004)

1980

1981

1982

1.4 Differentiate between, and use appropriate units of measures for, two- and three-dimensional objects (i.e., find the perimeter, area, volume).

1983

1984

**2.0 Students identify, describe, and classify the properties of, and the relationships between, plane and solid geometric figures:**

1985

1986

**2.1**

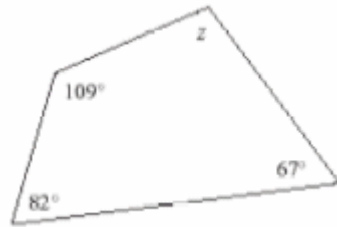
Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).

1987

1988

**2.2**

Know that the sum of the angles of any triangle is  $180^\circ$  and the sum of the angles of any quadrilateral is  $360^\circ$  and use this information to solve problems.



What is the measure of angle  $Z$  in the figure above? (CST released test question, 2004)

2.3 Visualize and draw two-dimensional views of three-dimensional objects made from rectangular solids.

---

## Statistics, Data Analysis, and Probability

**1.0 Students display, analyze, compare, and interpret different data sets, including data sets of different sizes:**

1.1 Know the concepts of mean, median, and mode; compute and compare simple examples to show that they may differ.

Compute the mean, median, and mode of the following collection of 27 numbers:

$\underbrace{1, 1, \dots, 1}_{23}, 1, 2, 3, 26, 135$

1.2 Organize and display single-variable data in appropriate graphs and representations (e.g., histogram, circle graphs) and explain which types of graphs are appropriate for various data sets.

1.3 Use fractions and percentages to compare data sets of different sizes.

2008 **1.4** Identify ordered pairs of data from a graph and interpret the meaning  
 2009 of the data in terms of the situation depicted by the graph.

2010 **1.5** Know how to write ordered pairs correctly; for example,  $(x, y)$ .

---

## 2011 **Mathematical Reasoning**

### 2012 **1.0 Students make decisions about how to approach problems:**

2013 1.1 Analyze problems by identifying relationships, distinguishing relevant  
 2014 from irrelevant information, sequencing and prioritizing information,  
 2015 and observing patterns.

2016 1.2 Determine when and how to break a problem into simpler parts.

### 2017 **2.0 Students use strategies, skills, and concepts in finding solutions:**

2018 2.1 Use estimation to verify the reasonableness of calculated results.

2019 2.2 Apply strategies and results from simpler problems to more complex  
 2020 problems.

2021 2.3 Use a variety of methods, such as words, numbers, symbols, charts,  
 2022 graphs, tables, diagrams, and models, to explain mathematical  
 2023 reasoning.

2024 2.4 Express the solution clearly and logically by using the appropriate  
 2025 mathematical notation and terms and clear language; support  
 2026 solutions with evidence in both verbal and symbolic work.

2027 2.5 Indicate the relative advantages of exact and approximate solutions to  
 2028 problems and give answers to a specified degree of accuracy.



2029	2.6	Make precise calculations and check the validity of the results from the
2030		context of the problem.

2031 **3.0 Students move beyond a particular problem by generalizing to other**  
2032 **situations:**

2033	3.1	Evaluate the reasonableness of the solution in the context of the
2034		original situation.

2035            3.2    Note the method of deriving the solution and demonstrate a  
2036                      conceptual understanding of the derivation by solving similar  
2037                      problems.

2038	3.3	Develop generalizations of the results obtained and apply them in
2039		other circumstances.

## 2040 **Grade Six Mathematics Contents Standards**

2041 By the end of grade six, students have mastered the four arithmetic operations  
 2042 with whole numbers, positive fractions, positive decimals, and positive and  
 2043 negative integers; they accurately compute and solve problems. They apply their  
 2044 knowledge to statistics and probability. Students understand the concepts of  
 2045 mean, median, and mode of data sets and how to calculate the range. They  
 2046 analyze data and sampling processes for possible bias and misleading  
 2047 conclusions; they use addition and multiplication of fractions routinely to calculate  
 2048 the probabilities for compound events. Students conceptually understand and  
 2049 work with ratios and proportions; they compute percentages (e.g., tax, tips,  
 2050 interest). Students know about  $\pi$  and the formulas for the circumference and area  
 2051 of a circle. They use letters for numbers in formulas involving geometric shapes  
 2052 and in ratios to represent an unknown part of an expression. They solve one-step  
 2053 linear equations.

---

### 2054 **Number Sense**

2055 **1.0 Students compare and order positive and negative fractions, decimals,**  
 2056 **and mixed numbers. Students solve problems involving fractions,**  
 2057 **ratios, proportions, and percentages:**

2058 **1.1** Compare and order positive and negative fractions, decimals, and  
 2059 mixed numbers and place them on a number line.

$$\frac{20}{21}, -\frac{4}{9}, -4.4, 1\frac{1}{12}, 1.1, -\frac{3}{7}.$$

2060 If you were to place  $-\frac{2}{3}$ ,  $-3$ , and  $-\frac{7}{8}$  on a number line, which number  
 2061 would be closest to  $-1$ ? Use a number line to explain your answer.

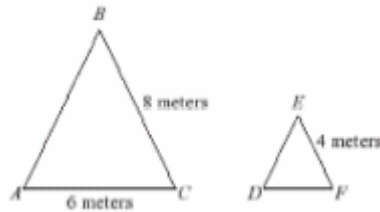
Place the following numbers on a number line:

$$0.3, -\frac{3}{10}, 2\frac{1}{2}, \frac{4}{5}, \frac{7}{8}, -2$$

**1.2** Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations ( $a/b$ ,  $a$  to  $b$ ,  $a:b$ ).

**1.3** Use proportions to solve problems (e.g., determine the value of  $N$  if  $4/7 = N/21$ , find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

$\triangle ABC$  is similar to  $\triangle DEF$ . What is the length of  $\overline{DF}$ ?



(CST released test question, 2004)

Ballpoint pens are sold in bundles of four. Lee bought 24 pens for \$14.40. How much would 56 pens cost? Carefully explain your solution.

Find  $n$  if:

$$1. \frac{49}{21} = \frac{14}{n}$$

$$2. \frac{n}{3} = \frac{5}{7}$$

(This problem also applies to Algebra and Functions Standard 1.1.)

**1.4** Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

Ann paid \$70.20 for a dress, and the amount includes an 8% sales tax. What is the cost of the dress before tax?

**2.0** Students calculate and solve problems involving addition, subtraction, multiplication, and division:

2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

Your after-school program is on a hiking trip. You hike  $\frac{3}{4}$  of a mile and stop to rest. Your friend hikes  $\frac{1}{5}$  of a mile, then turns around and hikes back  $\frac{1}{8}$  of a mile. Who is farther ahead on the trail? How much farther? Explain how you solved the problem.

At soccer practice the team has to run around a rectangular field that is  $75\frac{1}{2}$  feet by  $127\frac{3}{4}$  feet. The coach makes the team run around the field three times. How many total feet does the team member run? Explain how you solved this problem.

Mario wants to make half of his special no-bake cookie recipe. The recipe calls for  $1\frac{3}{4}$  cups of white sugar,  $\frac{1}{3}$  cup of margarine,  $\frac{1}{2}$  cup of peanut butter, and  $3\frac{1}{4}$  cups of oats. How much of each ingredient will Mario need? Explain how you solved this problem

Jim was on a hiking trail and after walking  $\frac{3}{4}$  of a mile, he found that he was only  $\frac{5}{8}$  of the way to the end of the trail. How long is the trail? Explain.

2.2 Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g.,  $\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \times \frac{16}{15} = \frac{2}{3}$ ).

1. If  $\frac{11}{7}$  is divided by a certain fraction  $\frac{a}{b}$ , the result is  $\frac{3}{8}$ . What is  $\frac{a}{b}$ ?

2. Draw a rectangle that has a perimeter of 1 and an area of less than  $\frac{1}{30}$ .

Draw a picture that illustrates each of the following problems and its solution. Explain how your drawings illustrate the problems and the solutions.

1.  $\frac{3}{4} \times \frac{1}{2}$

2.  $\frac{3}{4} \div \frac{1}{2}$

3.  $2 \times \frac{1}{4}$

**2.3** Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

Two friends start out on a daylong hike. They start at an elevation of 526 feet. The morning hike takes them to an altitude 300 feet higher than where they started. In the afternoon the friends descend 117 feet and stop to rest. Then they continue downward and descend another 366 feet. Describe the change in altitude.

$12 \div 3 = ?$

Simplify to make the calculation as simple as possible:

1.  $-19 + 37 + 19$

2127 2.  $(-16)(-28) + (-16)29$

2128 3.  $\left(-\frac{7}{8}\right)\left(\frac{17}{17}\right)$

2129 4.  $(-8)(-4)(19)(6 + (-6))$

2130 **2.4** Determine the least common multiple and the greatest common divisor  
 2131 of whole numbers; use them to solve problems with fractions (e.g., to  
 2132 find a common denominator to add two fractions or to find the reduced  
 2133 form for a fraction).  
 2134  $\frac{3}{8} + \frac{1}{12} = ?$  (CST released test question, 2004)

2135

---

2136 **Algebra and Functions**

2137 **1.0 Students write verbal expressions and sentences as algebraic**  
 2138 **expressions and equations; they evaluate algebraic expressions, solve**  
 2139 **simple linear equations, and graph and interpret their results:**

2140 **1.1** Write and solve one-step linear equations in one variable.

2141 What value of  $k$  makes the following equation true?

2142  $k \div 3 = 36$  (CST released test question, 2004)

2143  $y - 2 = 10$ . What is  $y$ ?

2144  $6y = 12$ . What is  $y$ ?

2145 If a number  $Y$  satisfies  $Y + 17 = 10$ , what is  $Y$ ? If a number  $X$  satisfies

2146  $3X = 25$ , what is  $X$ ?

2147

2148 **1.2** Write and evaluate an algebraic expression for a given situation, using  
 2149 up to three variables.

2150

2151

A telephone company charges \$0.05 per minute for local calls and

2152

\$0.12 per minute for long-distance calls. Which expression gives the

2153

total cost in dollars for X minutes of local calls and Y minutes of long-

2154

distance calls? (CST released test question, 2004)

2155

 $0.05X + 0.12y$        $0.05x - 0.12y$        $0.17(x + y)$        $0.17xy$ 

2156

1.3 Apply algebraic order of operations and the commutative, associative,

2157

and distributive properties to evaluate expressions; and justify each

2158

step in the process.

2159

Simplify:

2160

1.  $(4^3 + 7) - (5 - 8)^3$ 

2161

2.  $11[5(7^2) - 3^2 - 12(20 + 5.4 + 2)]$ 

2162

3.  $-3 \cdot (3^2 + 3) \div 3^2$ 

2163

1.4 Solve problems manually by using the correct order of operations or by

2164

using a scientific calculator.

2165

**2.0 Students analyze and use tables, graphs, and rules to solve problems**

2166

**involving rates and proportions:**

2167

2.1 Convert one unit of measurement to another (e.g., from feet to miles,

2168

from centimeters to inches).

2169

Suppose that one British pound is worth \$1.50. In London a magazine

2170

costs 3 pounds. In San Francisco the same magazine costs \$4.25. In

2171

which city is the magazine cheaper?

2172

When temperature is measured in both Celsius (C) and Fahrenheit (F),

2173

it is known that they are related by the following formula:

2174  $9 \times C = (F - 32) \times 5$ . What is 50 degrees Fahrenheit in Celsius?  
 2175 (Note the explicit use of parentheses.)  
 2176 How many inches are in  $2 \frac{1}{2}$  feet? (CST released test question, 2004)

2177 **2.2** Demonstrate an understanding that *rate* is a measure of one quantity  
 2178 per unit value of another quantity.

2179 Joe can type 9 words in 8 seconds. At this rate, how many words can  
 2180 he type in 2 minutes?

2181 **2.3** Solve problems involving rates, average speed, distance, and time.

2182 Marcus took a train from San Francisco to San Jose, a distance of  
 2183 54 miles. The train took 45 minutes for the trip. What was the average  
 2184 speed of the train?

2185 **3.0 Students investigate geometric patterns and describe them**  
 2186 **algebraically:**

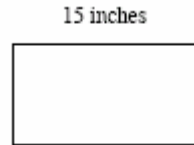
2187 **3.1** Use variables in expressions describing geometric quantities  
 2188 (e.g.,  $P = 2w + 2l$ ,  $A = \frac{1}{2}bh$ ,  $C = \pi d$ —the formulas for the perimeter of  
 2189 a rectangle, the area of a triangle, and the circumference of a circle,  
 2190 respectively).

2191 A rectangle has width  $w$ . Its length is one more than 3 times its width.  
 2192 Find the perimeter of the rectangle. (Your answer will be expressed in  
 2193 terms of  $w$ .)

2194 **3.2** Express in symbolic form simple relationships arising from geometry.  
 2195 The rectangle shown below has length 15 inches and perimeter  $p$   
 2196 inches.



2197



2198

2199 Which equation could be used to find the width of the rectangle?

2200  $P = 15 + w/2$        $P = 15 - w$        $P = 30 + 2w$        $P = 30 - 2w$ 

2201 (CST released test question, 2004)

2202 **Measurement and Geometry**

2203 **1.0 Students deepen their understanding of the measurement of plane and**  
 2204 **solid shapes and use this understanding to solve problems:**

**1.1**

Understand the concept of a constant such as  $\pi$ ; know the formulas for  
 the circumference and area of a circle.

Which equation could be used to find the area in square  
 inches of a circle with a radius of 8 inches?

(CST released

$$A = 4 \times \pi \quad A = \pi \times 4^2 \quad A = 8 \times \pi \quad A = \pi \times 8^2$$

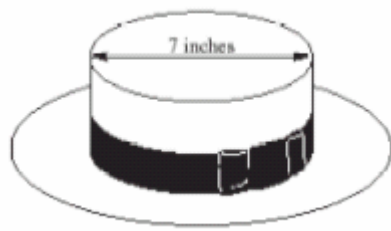
test question, 2004)

1.2 Know common estimates of  $\pi$  (3.14;  $\frac{22}{7}$ ) and use these values to  
 estimate and calculate the circumference and the area of circles;  
 compare with actual measurements.

What is the circumference of a circle with a radius of 5?

(Answer: 10 pi or approximately 31.4.)

The top part of this hat is shaped like a cylinder with a diameter of 7  
 inches.



2216

2217

Which measure is *closest* to the length of the band that goes around the outside of the hat? (CST released test question, 2004)

2218

2219

10.1 inches    11.0 inches    22.0 inches    38.5 inches

2220

- 1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base  $\times$  height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.

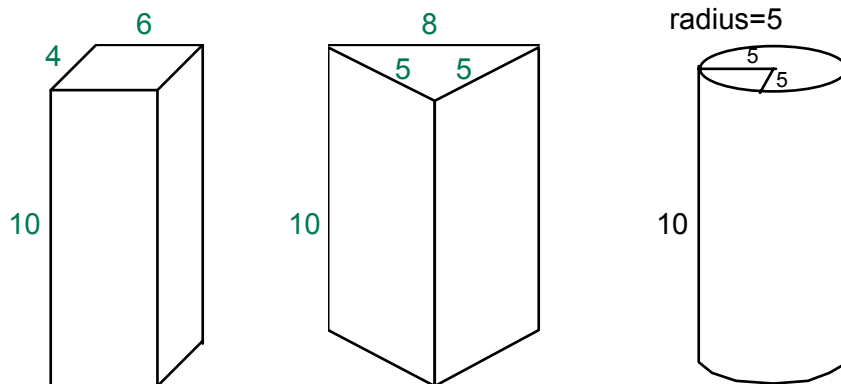
2221

2222

2223

2224

Find the volumes (dimensions are in cm).



2225

**2.0 Students identify and describe the properties of two-dimensional figures:**

2226

2227

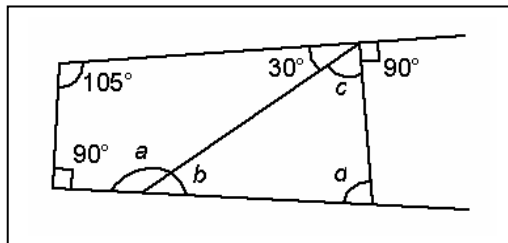
- 2.1 Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.

2228

**2.2**

Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.

Find the missing angles  $a$ ,  $b$ ,  $c$ , and  $d$ .



2.3 Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).

---

## **Statistics, Data Analysis, and Probability**

### **1.0 Students compute and analyze statistical measurements for data sets:**

1.1 Compute the range, mean, median, and mode of data sets.

1.2 Understand how additional data added to data sets may affect these computations of measures of central tendency.

1.3 Understand how the inclusion or exclusion of outliers affects measures of central tendency.

1.4 Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.

### **2.0 Students use data samples of a population and describe the characteristics and limitations of the samples:**

2.1 Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.

2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

2.3 Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.

2.4 Identify data that represent sampling errors and explain why the sample (and the display) might be biased.

2.5 Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

Calvin has been identified as the best runner in your school because he won the fifty-yard dash at the all-schools track meet. Use the records of the track team shown in the table below to decide if Calvin is the best runner in the school. Explain your decision, using the data.

<i>Runner</i>	<i>Race 1</i>	<i>Race 2</i>	<i>Race 3</i>	<i>Race 4</i>
Brian	27.3	27.6	30.1	26.2
Maria	26.5	26.3	26.0	27.1
Calvin	30.2	28.1	29.4	25.0
Alice	28.2	29.0	32.0	27.4
Fred	32.1	32.5	29.0	30.0
José	26.2	26.0	25.8	25.5

Soraya has been assigned to do a survey for the student council.

However, she forgets to do this until the morning of the meeting, so she asks three of her best friends what kind of music they would like for a

2268                    noon-time dance. Their opinions are what Soraya will report to student  
2269                    council.

2270                    Do you think Soraya's report is an accurate reflection of the kind of  
2271                    music that students want played for the noon-time dance? Explain  
2272                    your answer.

2273    **3.0   Students determine theoretical and experimental probabilities and use**  
2274                    **these to make predictions about events:**

2275    **3.1**   Represent all possible outcomes for compound events in an organized way  
2276                    (e.g., tables, grids, tree diagrams) and express the theoretical probability of  
2277                    each outcome.

2278    **3.2**   Use data to estimate the probability of future events (e.g., batting averages  
2279                    or number of accidents per mile driven).

2280    **3.3**   Represent probabilities as ratios, proportions, decimals between 0 and 1,  
2281                    and percentages between 0 and 100 and verify that the probabilities  
2282                    computed are reasonable; know that if  $P$  is the probability of an event,  $1-P$  is  
2283                    the probability of an event not occurring.

2284    **3.4**   Understand that the probability of either of two disjoint events occurring is  
2285                    the sum of the two individual probabilities and that the probability of one  
2286                    event following another, in independent trials, is the product of the two  
2287                    probabilities.

2288                    Understand the difference between independent and dependent  
2289                    events.

---

2290    **Mathematical Reasoning**

2291    **1.0   Students make decisions about how to approach problems:**

- 2292 1.1 Analyze problems by identifying relationships, distinguishing relevant  
2293 from irrelevant information, identifying missing information, sequencing  
2294 and prioritizing information, and observing patterns.
- 2295 1.2 Formulate and justify mathematical conjectures based on a general  
2296 description of the mathematical question or problem posed.
- 2297 1.3 Determine when and how to break a problem into simpler parts.
- 2298 **2.0 Students use strategies, skills, and concepts in finding solutions:**
- 2299 2.1 Use estimation to verify the reasonableness of calculated results.
- 2300 2.2 Apply strategies and results from simpler problems to more complex  
2301 problems.
- 2302 2.3 Estimate unknown quantities graphically and solve for them by using  
2303 logical reasoning and arithmetic and algebraic techniques.
- 2304 2.4 Use a variety of methods, such as words, numbers, symbols, charts,  
2305 graphs, tables, diagrams, and models, to explain mathematical  
2306 reasoning.
- 2307 2.5 Express the solution clearly and logically by using the appropriate  
2308 mathematical notation and terms and clear language; support  
2309 solutions with evidence in both verbal and symbolic work.
- 2310 2.6 Indicate the relative advantages of exact and approximate solutions to  
2311 problems and give answers to a specified degree of accuracy.
- 2312 2.7 Make precise calculations and check the validity of the results from the  
2313 context of the problem.

2314 **3.0 Students move beyond a particular problem by generalizing to other**  
2315 **situations:**

2316 3.1 Evaluate the reasonableness of the solution in the context of the  
2317 original situation.

2318 3.2 Note the method of deriving the solution and demonstrate a  
2319 conceptual understanding of the derivation by solving similar  
2320 problems.

2321 3.3 Develop generalizations of the results obtained and the strategies  
2322 used and apply them in new problem situations.

## 2323 **Grade Seven Mathematics Content Standards**

2324 By the end of grade seven, students are adept at manipulating numbers and  
 2325 equations and understand the general principles at work. Students understand  
 2326 and use factoring of numerators and denominators and properties of exponents.  
 2327 They know the Pythagorean theorem and solve problems in which they compute  
 2328 the length of an unknown side. Students know how to compute the surface area  
 2329 and volume of basic three-dimensional objects and understand how area and  
 2330 volume change with a change in scale. Students make conversions between  
 2331 different units of measurement. They know and use different representations of  
 2332 fractional numbers (fractions, decimals, and percents) and are proficient at  
 2333 changing from one to another. They increase their facility with ratio and  
 2334 proportion, compute percents of increase and decrease, and compute simple and  
 2335 compound interest. They graph linear functions and understand the idea of slope  
 2336 and its relation to ratio.

---

## 2337 **Number Sense**

2338 **1.0 Students know the properties of, and compute with, rational numbers**  
 2339 **expressed in a variety of forms:**

2340 1.1 Read, write, and compare rational numbers in scientific notation  
 2341 (positive and negative powers of 10), compare rational numbers in  
 2342 general.

2343 Put the following numbers on the number line:

2344  $-3.14$ ,  $-3.3$ ,  $-3\frac{1}{3}$ ,  $-3.1$ ,  $-\frac{27}{8}$

2345 Arrange in increasing order the following numbers:

2346  $1.86 \times 10^5$ , 185,766,  $1.004 \times 10^6$ ,  $2.1 \times 10^5$ , 205,666



2347 Arrange in increasing order the following numbers:

2348  $-3.14 \times 10^2$      $3.14 \times 10^2$      $-3.14 \times 10^2$      $3.14 \times 10^{-2}$

2349

2350 **1.2** Add, subtract, multiply, and divide rational numbers (integers,  
2351 fractions, and terminating decimals) and take positive rational numbers  
2352 to whole-number powers.

2353 1.  $\frac{1}{4} \times 0.33$

2354 2.  $\frac{2\frac{1}{7}}{\frac{2}{3}} - \frac{\frac{3}{7}}{\left(\frac{2}{3}\right)^2}$

2355 3. Evaluate:

2356  $\frac{12}{7} \times \frac{6}{5} \times \frac{7}{8} =$

2357  $3\frac{1}{5} \div (-5) =$

2358  $(0.2)^5 \times \left(\frac{3}{2}\right)^4 =$

2359  $\frac{1}{2}(58.3 - 11.29) =$

2360 1.3 Convert fractions to decimals and percents and use these  
2361 representations in estimations, computations, and applications.

2362 Change to decimals:

2363  $\frac{7}{8}$      $\frac{7}{11}$

2364 **1.4** Differentiate between rational and irrational numbers.

2365 Which is an irrational number? (CST released test question, 2004)

2366  $\sqrt{5}$      $\sqrt{9}$      $-1$      $-2/3$

**1.5** Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.

Change to fractions:

0.25       $\overline{0.27}$

Find the period of the repeating part of  $\frac{41}{13}$ .

**1.6** Calculate the percentage of increases and decreases of a quantity.  
A sweater originally cost \$37.50. Last week, Moesha bought it at 20% off. (CST released test question, 2004)



How much was deducted from the original price?

A. \$7.50      B. \$17.50      C. \$20.00      D. \$30.00

**1.7** Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

Hung-Hsi deposits \$800 in an account that earns 10% (simple) interest. Jim deposits \$800 in an account that earns 10% interest compounded yearly. Who will have more money at the end of one years, two years, and three years? Who will have more money over the long run? Explain why.

Jason bought a jacket on sale for 50% off the original price and

2388 another 25% off the discounted price. If the jacket originally cost \$88,  
 2389 what was the final sale price that Jason paid for the jacket? (CST  
 2390 released test question, 2004)

2391 **2.0 Students use exponents, powers, and roots and use exponents in**  
 2392 **working with fractions:**

2393 2.1 Understand negative whole-number exponents. Multiply and divide  
 2394 expressions involving exponents with a common base.

2395 Simplify:

2396 1. 
$$\frac{\left(\frac{2}{7}\right)^5 \times \left(\frac{2}{7}\right)^{11}}{\left(\frac{2}{7}\right)^3}$$

2397 2. 
$$\left(\frac{2}{3}\right)^{-3} \times \frac{2}{9}$$

2398 **2.2** Add and subtract fractions by using factoring to find common  
 2399 denominators.

2400 Make use of prime factors to compute:

2401 1. 
$$\frac{2}{28} + \frac{1}{49}$$

2402 2. 
$$\frac{-5}{63} + \left(\frac{-7}{99}\right)$$

2403 **2.3** Multiply, divide, and simplify rational numbers by using exponent rules.

2404 Simplify:

2405 1. 
$$\frac{\left(-\frac{2}{3}\right)^3}{2\frac{1}{4}} + \left(\frac{3}{-2}\right)^2 \left(4 - 3\frac{1}{3}\right)$$

2406 2.  $\frac{\left(\frac{2}{5} \times 2\frac{1}{3}\right)^4}{\left(\frac{2}{5}\right)\left(-2\frac{1}{3}\right)^3}$

2407 3.  $\frac{3^{-2}}{2^{-3}}$

2408 4.  $\frac{2x^3}{2^3x^{-1}}$

2409 5.  $\frac{4^2 \cdot 3^5 \cdot 2^4}{4^3 \cdot 3^5 \cdot 2^2} =$  (CST released test question, 2004)

2410 2.4 Use the inverse relationship between raising to a power and extracting  
 2411 the root of a perfect square integer; for an integer that is not square,  
 2412 determine without a calculator the two integers between which its  
 2413 square root lies and explain why.

2414 Find the length of one side of a square which has an area of 81.

2415 **2.5** Understand the meaning of the absolute value of a number; interpret  
 2416 the absolute value as the distance of the number from zero on a  
 2417 number line; and determine the absolute value of real numbers.

2418 Is it always true that for any numbers  $a$  and  $b$ ,  $a - |b| \leq a + b$ ?

2419  $|9 - 5| - |6 - 8| = ?$  (CST released test question, 2004)

## 2420 Algebra and Functions

2421 **1.0 Students express quantitative relationships by using algebraic**  
 2422 **terminology, expressions, equations, inequalities, and graphs:**

2423 1.1 Use variables and appropriate operations to write an expression, an  
 2424 equation, an inequality, or a system of equations or inequalities that  
 2425 represents a verbal description (e.g., three less than a number, half as  
 2426 large as area A).

2427 Write the following verbal statements as algebraic expressions:

2428 1. The square of  $a$  is increased by the sum of twice  $a$  and 3.

2429 2. The product of  $\frac{1}{2}$  of  $a$  and 3 is decreased by the quotient of  $a$   
2430 divided by  $(-4)$ .

2431 1.2 Use the correct order of operations to evaluate algebraic expressions  
2432 such as  $3(2x + 5)^2$ .

2433 Given  $x = (-2)$  and  $y = 5$  evaluate:

2434 1.  $x^2 + 2x - 3$

2435 2.  $\frac{y(xy - 7)}{10}$

2436 **1.3** Simplify numerical expressions by applying properties of rational  
2437 numbers (e.g., identity, inverse, distributive, associative, commutative)  
2438 and justify the process used.

2439 Name the property illustrated by each of the following:

2440 1.  $x(y + -y) = x(0)$

2441 2.  $x(y + -y) = xy + x(-y)$

2442 3.  $x(y + -y) = (y + -y)(x)$

2443 4.  $x(y + -y) = x(-y + y)$

2444 5.  $x(y(1/y)) = x(1)$

2445 1.4 Use algebraic terminology (e.g., variable, equation, term, coefficient,  
2446 inequality, expression, constant) correctly.

2447 1.5 Represent quantitative relationships graphically and interpret the  
2448 meaning of a specific part of a graph in the situation represented by  
2449 the graph.

2450 **2.0 Students interpret and evaluate expressions involving integer powers**  
2451 **and simple roots:**

2452 2.1 Interpret positive whole-number powers as repeated multiplication and  
2453 negative whole-number powers as repeated division or multiplication  
2454 by the multiplicative inverse. Simplify and evaluate expressions that  
2455 include exponents.

2456 2.2 Multiply and divide monomials; extend the process of taking powers  
2457 and extracting roots to monomials when the latter results in a  
2458 monomial with an integer exponent.

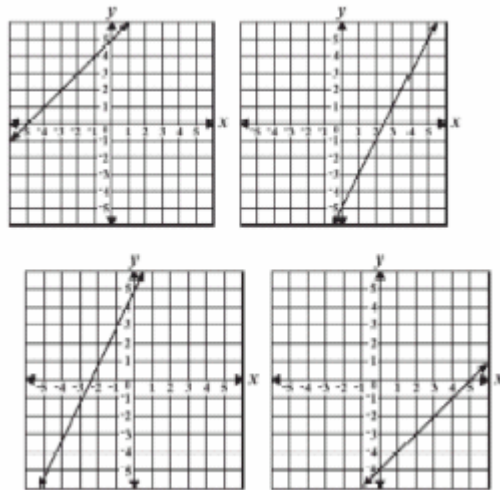
2459 **3.0 Students graph and interpret linear and some nonlinear functions:**

2460 3.1 Graph functions of the form  $y = nx^2$  and  $y = nx^3$  and use in solving  
2461 problems.

2462 3.2 Plot the values from the volumes of three-dimensional shapes for  
2463 various values of the edge lengths (e.g., cubes with varying edge  
2464 lengths or a triangle prism with a fixed height and an equilateral  
2465 triangle base of varying lengths).

2466 **3.3** Graph linear functions, noting that the vertical change (change in  $y$ -  
2467 value) per unit of horizontal change (change in  $x$ -value) is always the  
2468 same and know that the ratio (“rise over run”) is called the slope of a  
2469 graph.

2470 A function of  $x$  has value 7 when  $x = 1$ ; it has value 15.5 when  $x = 3.5$ ;  
 2471 and it has value 20 when  $x = 5$ . Is this a linear function?  
 2472 Which best represents the graph of  $y=2x-5$ ? (CST released test  
 2473 question, 2004)



2474

**3.4** Plot the values of quantities whose ratios are always the same (e.g.,  
 2475 cost to the number of an item, feet to inches, circumference to  
 2476 diameter of a circle). Fit a line to the plot and understand that the slope  
 2477 of the line equals the quantities.  
 2478 of the line equals the quantities.

2479 **4.0** Students solve simple linear equations and inequalities over the  
 2480 rational numbers:

2481 **4.1** Solve two-step linear equations and inequalities in one variable over  
 2482 the rational numbers, interpret the solution or solutions in the context  
 2483 from which they arose, and verify the reasonableness of the results.

2484 Solve for  $x$  if  $3x-12 = 3,821$ . If  $x$  stands for the number of books in a  
 2485 bookstore, can it satisfy this equation?

2486

2487 What is the solution set to the inequality  $6z + 5 > 35$ ? (CST released  
 2488  $\{z : z < 5\}$   $\{z : z < 24\}$   $\{z : z > 5\}$   $\{z : z > 24\}$  test question, 2004)

2489 **4.2** Solve multistep problems involving rate, average speed, distance, and  
 2490 time or a direct variation.

2491 A train can travel at either of two speeds between two towns that are  
 2492 72 miles apart. The higher speed is 25% faster than the lower speed  
 2493 and reduces the travel time by 30 minutes. What are the two speeds in  
 2494 miles per hour.

2495 A duck flew at 18 miles per hour for 3 hours, then at 15 miles per hour  
 2496 for 2 hours. How far did the duck fly in all? (CST released test  
 2497 question, 2004)

2498 Juanita earns \$36 for 3 hours of work. At that rate, how long would she  
 2499 have to work to earn \$720? (CST released test question, 2004)

## 2500 **Measurement and Geometry**

2501 **1.0 Students choose appropriate units of measure and use ratios to**  
 2502 **convert within and between measurement systems to solve problems:**

2503 1.1 Compare weights, capacities, geometric measures, times, and  
 2504 temperatures within and between measurement systems (e.g., miles  
 2505 per hour and feet per second, cubic inches to cubic centimeters).

2506 Convert the following:

2507 1. 80 miles/hr. = ? ft./sec.

2508 2. 20 oz./min. = ? qts./day

2509 1.2 Construct and read drawings and models made to scale.



**1.3**

Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

Printer	Description
Roboprint	Prints 2 pages per second
Volttronn	Prints 1 page every 2 seconds
Vantek Plus	Prints 160 pages in 2 minutes
DLS Pro	Prints 100 pages per minute

Which printer is fastest?

(CST released test question, 2004)

**2.0 Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:**

2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

2.2 Estimate and compute the area of more complex or irregular two- and three-dimensional figures by breaking the figures down into more basic geometric objects.

2.3 Compute the length of the perimeter, the surface area of the faces, and the volume of a three-dimensional object built from rectangular solids. Understand that when the lengths of all dimensions are multiplied by a scale factor, the surface area is multiplied by the

2530 square of the scale factor and the volume is multiplied by the cube of  
2531 the scale factor.

2532 2.4 Relate the changes in measurement with a change of scale to the  
2533 units used (e.g., square inches, cubic feet) and to conversions  
2534 between units (1 square foot = 144 square inches or  $[1 \text{ ft}^2] = [144 \text{ in}^2]$ ,  
2535 1 cubic inch is approximately 16.38 cubic centimeters or  $[1 \text{ in}^3] =$   
2536  $[16.38 \text{ cm}^3]$ ).

2537 **3.0 Students know the Pythagorean theorem and deepen their**  
2538 **understanding of plane and solid geometric shapes by constructing**  
2539 **figures that meet given conditions and by identifying attributes of**  
2540 **figures:**

2541 3.1 Identify and construct basic elements of geometric figures (e.g.,  
2542 altitudes, midpoints, diagonals, angle bisectors, and perpendicular  
2543 bisectors; central angles, radii, diameters, and chords of circles) by  
2544 using a compass and straightedge.

2545 3.2 Understand and use coordinate graphs to plot simple figures,  
2546 determine lengths and areas related to them, and determine their  
2547 image under translations and reflections.

2548 **3.3** Know and understand the Pythagorean theorem and its converse and  
2549 use it to find the length of the missing side of a right triangle and the  
2550 lengths of other line segments and, in some situations, empirically  
2551 verify the Pythagorean theorem by direct measurement.

2552 What is the side length of an isosceles right triangle with hypotenuse  
2553  $\sqrt{72}$ ?

A right triangle has sides of lengths  $a$ ,  $b$ , and  $c$ ;  $c$  is the length of the hypotenuse. How would the areas of the three equilateral triangles with sides of lengths  $a$ ,  $b$ ,  $c$ , respectively, be related to each other?

**3.4** Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

**3.5** Construct two-dimensional patterns for three-dimensional models, such as cylinders, prisms, and cones.

**3.6** Identify elements of three-dimensional geometric objects (e.g., diagonals of rectangular solids) and describe how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect).

True or false? If true, give an example. If false, explain why.

Two planes in three-dimensional space can:

1. Intersect in a line.
2. Intersect in a single point.
3. Have no intersection at all.

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## Statistics, Data Analysis, and Probability

**1.0** Students collect, organize, and represent data sets that have one or more variables and identify relationships among variables within a data set by hand and through the use of an electronic spreadsheet software program:

1.1 Know various forms of display for data sets, including a stem-and-leaf plot or box-and-whisker plot; use the forms to display a single set of data or to compare two sets of data.

1.2 Represent two numerical variables on a scatterplot and informally describe how the data points are distributed and any apparent relationship that exists between the two variables (e.g., between time spent on homework and grade level).

**1.3** Understand the meaning of, and be able to compute, the minimum, the lower quartile, the median, the upper quartile, and the maximum of a data set.

Here is a set of data for an exam in a mathematics class:

Minimum	45
Lower quartile score	51
Median	64
Upper quartile score	72
Maximum	92

(a) Suppose there are 15 students in the class. Give a range of scores that would satisfy all the data shown above.

(b) Suppose 7 students have scores ranging from 64 to 72. How many students might there be in the class? Explain.

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## Mathematical Reasoning

**1.0 Students make decisions about how to approach problems:**

- 2598           1.1   Analyze problems by identifying relationships, distinguishing relevant  
2599                   from irrelevant information, identifying missing information, sequencing  
2600                   and prioritizing information, and observing patterns.
- 2601           1.2   Formulate and justify mathematical conjectures based on a general  
2602                   description of the mathematical question or problem posed.
- 2603           1.3   Determine when and how to break a problem into simpler parts.
- 2604   **2.0   Students use strategies, skills, and concepts in finding solutions:**
- 2605           2.1   Use estimation to verify the reasonableness of calculated results.
- 2606           2.2   Apply strategies and results from simpler problems to more complex  
2607                   problems.
- 2608           2.3   Estimate unknown quantities graphically and solve for them by using  
2609                   logical reasoning and arithmetic and algebraic techniques.
- 2610           2.4   Make and test conjectures by using both inductive and deductive  
2611                   reasoning.
- 2612           2.5   Use a variety of methods, such as words, numbers, symbols, charts,  
2613                   graphs, tables, diagrams, and models, to explain mathematical  
2614                   reasoning.
- 2615           2.6   Express the solution clearly and logically by using the appropriate  
2616                   mathematical notation and terms and clear language; support  
2617                   solutions with evidence in both verbal and symbolic work.
- 2618           2.7   Indicate the relative advantages of exact and approximate solutions to  
2619                   problems and give answers to a specified degree of accuracy.

2620           2.8    Make precise calculations and check the validity of the results from the  
2621                   context of the problem.

2622   **3.0   Students determine a solution is complete and move beyond a**  
2623   **particular problem by generalizing to other situations:**

2624           3.1    Evaluate the reasonableness of the solution in the context of the  
2625                   original situation.

2626           3.2    Note the method of deriving the solution and demonstrate a  
2627                   conceptual understanding of the derivation by solving similar  
2628                   problems.

2629           3.3    Develop generalizations of the results obtained and the strategies  
2630                   used and apply them to new problem situations.